The Constants of the Physical Libration of the Moon.

By F. J. M. Stratton, M.A.

(*Abstract.*)

The following are the results obtained from a fresh reduction of the observations of Mösting A made by Schlüter at Königsberg in the years 1841–3. The observed positions were reduced so as to give the apparent selenographical latitude (β) and longitude (λ) of the crater at each observation. Two reductions were undertaken: one was based on the previous work of Dr. Franz on these observations; in the second reduction an assumption of Dr. Franz with regard to the constancy of the focal setting of the heliometer was not adopted. One element, which had been overlooked by Dr. Franz, was taken into account in both reductions, namely, the excess (dh) of the radius of the moon to Mösting A over the observed mean radius to the limb.

Two solutions were made for each reduction. In the one case, the *unrestricted* case, the residuals between the observed β, λ and an assumed constant β, λ were analysed by the method of least squares for corrections to the assumed constants, and for certain periodic terms; no theoretical connection between the coefficients of these periodic terms was assumed in the analysis. The value of one of the unknown constants $\left(f \equiv \dfrac{(\mathrm{C}-\mathrm{B})\,\mathrm{B}}{(\mathrm{C}-\mathrm{A})\,\mathrm{A}}\right)$ was then derived from the coefficients of these periodic terms. In the second or *restricted* solution, a connection between the coefficients was assumed beforehand, according to the scheme worked out by Dr. Hayn,* and the values of f and of the other constants were obtained on this theoretical basis.

The following four solutions were thus obtained:—

1. Unrestricted solutions.

(*a*) With Dr. Franz's assumption:

From Latitude Equations.	From Longitude Equations.
$\beta = -3^\circ\,9'\,36'' \pm 47''$	$\lambda = -5^\circ\,9'\,18'' \pm 42''$
$\mathrm{I} = 1^\circ\,28'\,39'' \pm 70''$	$\mathrm{I} = 1^\circ\,35'\,2'' \pm 206''$
$f = 0{\cdot}51 \pm 0{\cdot}09$	$f = 0{\cdot}38 \pm 0{\cdot}04$
$dh = +8''{\cdot}8 \pm 2''{\cdot}2$	$dh = +1''{\cdot}9 \pm 2''{\cdot}2$
$\delta\beta = -106''\sin\omega - 17''\cos\omega - 77''\sin(g+\omega-\lambda)$	$\delta\lambda = -66''\sin g + 168''\sin g' - 26''\sin 2\omega$
p.e. of observation $= 61''$	p.e. of observation $= 89''$

* *Selenographische Koordinaten*, ii. p. 52.

(*b*) Without Dr. Franz's assumption:

From Latitude Equations.

$\beta = -3^\circ\ 10'\ 2'' \pm 54''$
$\mathrm{I} = 1^\circ\ 28'\ 34'' \pm 81''$
$f = 0{\cdot}66 \pm 0{\cdot}11$
$dh = +8''{\cdot}2 \pm 3''{\cdot}1$
$\delta\beta = -125''\sin\omega - 23''\cos\omega - 50''\sin(g+\omega-\lambda)$

p.e. of observation $= 69''$

From Longitude Equations.

$\lambda = -5^\circ\ 7'\ 1'' \pm 58''$
$\mathrm{I} = 1^\circ\ 34'\ 8'' \pm 281''$
$f = 0{\cdot}50 \pm 0{\cdot}05$
$dh = +5''{\cdot}0 \pm 3''{\cdot}0$
$\delta\lambda = +20''\sin g + 148''\sin g' - 51''\sin 2\omega$

p.e of observation $= 121''$

2. Restricted solutions.

(*a*) With Dr. Franz's assumption:

From Latitude Equations.

$\beta = -3^\circ\ 9'\ 28'' \pm 40''$
$\mathrm{I} = 1^\circ\ 27'\ 31'' \pm 66''$
$f = 0{\cdot}49 \pm 0{\cdot}03$
$dh = +7''{\cdot}9 \pm 2''{\cdot}5$
$\delta\beta = -93''\sin\omega + 7''\cos\omega + 11''\sin(g+\omega-\lambda)$

p.e. of observation $= 61''$

From Longitude Equations.

$\lambda = -5^\circ\ 9'\ 0'' \pm 11''$
$\mathrm{I} = 1^\circ\ 33'\ 40'' \pm 207''$
$f = 0{\cdot}42 \pm 0{\cdot}03$
$dh = 3''{\cdot}0 \pm 0''{\cdot}5$
$\delta\lambda = -25''\sin g + 154''\sin g' - 14''\sin 2\omega$

p.e. of observation $= 90''$

(*b*) Without Dr. Franz's assumption:

From Latitude Equations.

$\beta = -3^\circ\ 10'\ 40'' \pm 46''$
$\mathrm{I} = 1^\circ\ 29'\ 19'' \pm 77''$
$f = 0{\cdot}60 \pm 0{\cdot}07$
$dh = +4''{\cdot}0 \pm 3''{\cdot}0$
$\delta\beta = -114''\sin\omega + 7''\cos\omega + 11''\sin(g+\omega-\lambda)$

p.e. of observation $= 70''$

From Longitude Equations.

$\lambda = -5^\circ\ 8'\ 25'' \pm 16''$
$\mathrm{I} = 1^\circ\ 33'\ 20'' \pm 279''$
$f = 0{\cdot}46 \pm 0{\cdot}04$
$dh = +3''{\cdot}0 \pm 0''{\cdot}7$
$\delta\lambda = -24''\sin g + 141''\sin g' - 17''\sin 2\omega$

p.e. of observation $= 121''$

If we put $dh = 0$ in the equations giving the solution 2(*a*), we ought to get a solution closely resembling Dr. Franz's solution. We get in fact:

From Latitude Equations.

$\beta = -3^\circ\ 11'\ 30'' \pm 7''$
$\mathrm{I} = 1^\circ\ 30'\ 57'' \pm 8''$
$f = 0{\cdot}58 \pm 0{\cdot}05$
$\delta\beta = -110''\sin\omega + 7''\cos\omega + 11''\sin(g+\omega-\lambda)$

p.e. of observation $= 61''$

From Longitude Equations.

$\lambda = -5^\circ\ 9'\ 56'' \pm 6''$
$\mathrm{I} = 1^\circ\ 25'\ 52'' \pm 203''$
$f = 0{\cdot}43 \pm 0{\cdot}03$
$\delta\lambda = -25''\sin g + 151''\sin g' - 14''\sin 2\omega$

p.e. of observation $= 95''$

Combining the two sets of equations with weights 2·4466 and unity for the latitude and longitude equations respectively (the weights assigned by Dr. Franz), this solution gives :

$$\beta = -3^\circ\, 11'\, 23''$$
$$\lambda = -5^\circ\, 9'\, 58''$$
$$I = 1^\circ\, 30'\, 59''$$
$$f = 0{\cdot}48$$

a result in pretty close accordance with Dr. Franz's solution of the same observational material.

The final conclusion is that Dr. Franz's results in the chief points where they differ from Dr. Hayn's are confirmed by the present investigation. If the uncertain element dh, neglected by Dr. Franz, is taken into account an alternative solution may be substituted for Dr. Franz's, namely 2(*b*) above. Combining the two sets of constants there given, we have :

$$\beta = -3^\circ\, 10'\, 40'' \pm 46''$$
$$\lambda = -5^\circ\, 8'\, 25'' \pm 16''$$
$$I = 1^\circ\, 29'\, 37'' \pm 71''$$
$$f = 0{\cdot}50 \pm 0{\cdot}03$$
$$dh = +3''{\cdot}0 \pm 0''{\cdot}7$$
$$\delta\beta = -95''\sin\omega + 7''\cos\omega + 11''\sin(g + \omega - \lambda)$$
$$\delta\lambda = -22''\sin g + 129''\sin g' - 20''\sin 2\omega$$

The difference between the results derived from Schlüter's observations and those recently obtained by Dr. Hayn remains. But this work shows that the difference lies in the observations themselves rather than in the theoretical treatment. The great need at the present moment is for some fresh observational material continued over a period of several years and free from the errors of the limb which affect so adversely the measures of this and other investigations.

It should be added that part of the expense of checking the calculations was borne by a grant from the Government Grant Committee of the Royal Society.

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Aberration considered in relation to the two Star-Streams.

By A. S. Eddington, M.A., M.Sc.

The suggestion here considered with regard to the two star-streams which appear to exist throughout the sky was made to me by Mr. Stratton nearly two years ago, and recently and independently by Professor Turner.* I believe that the result of examining the data of the observed proper motions is to show that it is not a correct explanation of the phenomena; but it seems worth while to publish it, not only on account of its intrinsic interest, but because the published investigations of the systematic motions of the stars hardly afford a means of testing it, and it is only by reference to the original data that we are able to ascertain whether it is or is not accordant with the facts.

The suggestion (practically in Professor Turner's words) is as follows :—

(*a*) The Sun's motion in space gives rise to an aberration effect, which, however, is usually neglected. The displacement of the stars from this cause is constant, and we may as well use the displaced as the true places.

(*b*) But if the solar motion were to change, there would be a change of this aberration displacement. An *acceleration* of the Sun would produce, by aberration, changes in the apparent positions of the stars very similar to those which its *velocity* produces by displacement; with this difference, however, that the aberration effect would be independent of the stars' parallaxes, whereas the displacement effect depends on them directly.

(*c*) Were it not for this difference, the effects of the two vectors (α) displacement and (β) change of velocity would compound, according to the vector law, and be inseparable by observation.

(*d*) But the difference specified in (*b*) gives an independent existence to the two vectors; so that, in a sense, the Sun has two apparent velocities relative to the stars.

(*e*) Is this a possible explanation of the two star-streams? The attractiveness of the explanation lies in the accounting for the complete intermingling of the two streams. The duplicity is in fact removed from the stars to the Sun.

Consider, for example, in a universe in which the true motions of the stars are everywhere haphazard, the two groups of stars which are respectively (1) very near to the Sun and (2) very distant from it. For the former the displacement effect (which is proportional to the parallax) will be great in comparison with the aberration effect, so that the latter effect may be neglected. This group of stars will accordingly appear to form a drift, whose motion relative to the Sun would be parallel to and opposite to the true solar displacement. On the other hand, for the very distant stars the displacement effect will be vanishingly small; and assuming the aberration effect to be sensible, that will be the only change of

* *Monthly Notices*, lxix. p. 412, sec. 22.